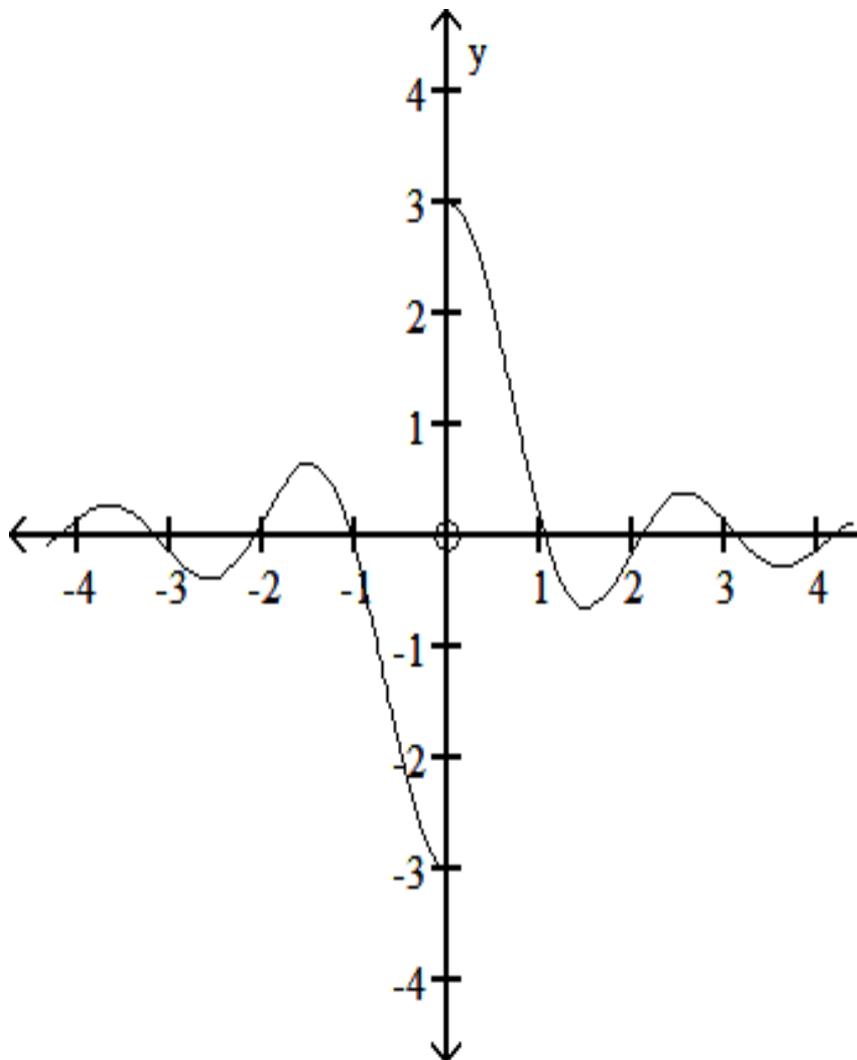Name_____

PLEASE SHOW ALL YOUR WORK AS NEATLY A S POSSIBLE :

SHOW ALL YOUR WORK TO RECEIVE FULL POINTS :

Use the graph to evaluate the limit. Does the limit exist ? Why?

1) $\lim\limits_{x \to 0^-} f(x) =$ $\lim\limits_{x \to 0^+} f(x) =$

_____

lim $x \to 0^-$ f(x)=-3
lim $x \to 0^+$ f(x)=3

The limit does not exist because there is a jump discontinuity at 0 i.e. left-hand limit ≠ right-hand limit.

2. Find the limits:

a) $\lim\limits_{x \to 0} \dfrac{x - sin(x)}{x}$

$$\lim\limits_{x \to 0} \left(\frac{x - sin(x)}{x} \right) = \lim\limits_{x \to 0} \left(\frac{\frac{d}{dx}(x - sin(x))}{\frac{d}{dx}(x)} \right)$$

$= \lim\limits_{x \to 0}(1 - cos(x))$

$= 1 - cos(0)$

$\lim\limits_{x \to 0} \dfrac{x - sin(x)}{x} = 0$

b) $\lim\limits_{x \to 0} \dfrac{sin(2x)}{2x^2 + x}$

$$\lim\limits_{x \to 0} \left(\frac{sin(2x)}{2x^2 + x} \right) = \lim\limits_{x \to 0} \left(\frac{\frac{d}{dx}(sin(2x))}{\frac{d}{dx}(2x^2 + x)} \right)$$

$= \lim\limits_{x \to 0} \left(\dfrac{2cos(2x)}{4x + 1} \right)$

$= \dfrac{2cos(0)}{4(0) + 1}$

$\lim\limits_{x \to 0} \dfrac{sin(2x)}{2x^2 + x} = 2$

3. Find the limits:

a) $\lim\limits_{x \to -\pi} \sqrt{x + 9}cos(x + \pi)$

$$= \sqrt{-\pi + 9} \cos(-\pi + \pi)$$

$$= \sqrt{-\pi + 9} \approx 2.42041$$

b) $\lim\limits_{x \to \infty} \dfrac{x - \cos(x)}{x}$

$$\lim\limits_{x \to \infty} \dfrac{x - \cos(x)}{x} = \lim\limits_{x \to \infty} (1 - \dfrac{\cos(x)}{x})$$

$$= \lim\limits_{x \to \infty} (1) - \lim\limits_{x \to \infty} (\dfrac{\cos(x)}{x})$$

But the limit of a constant is equal to the constant.

$$\dfrac{-1}{x} \le (\dfrac{\cos(x)}{x}) \le \dfrac{1}{x}$$

$$= \lim\limits_{x \to \infty} (\dfrac{-1}{x}) \le \lim\limits_{x \to \infty} (\dfrac{\cos(x)}{x}) \le \lim\limits_{x \to \infty} (\dfrac{1}{x})$$

$$0 \le (\dfrac{\cos(x)}{x}) \le 0$$

Thus $\lim\limits_{x \to \infty} (\dfrac{\cos(x)}{x}) = 0$

$$= 1 + 0$$

$$\lim\limits_{x \to \infty} \dfrac{x - \cos(x)}{x} = 1$$

4. Find the limits:

a) $\lim\limits_{x \to 0} \dfrac{1 - \cos(x)}{x^2}$

$$\lim\limits_{x \to 0} \left(\dfrac{1 - \cos(x)}{x^2} \right) = \lim\limits_{x \to 0} \left(\dfrac{\frac{d}{dx}(1 - \cos(x))}{\frac{d}{dx}(x^2)} \right)$$

$$= \lim\limits_{x \to 0} \left(\dfrac{\sin(x)}{2x} \right)$$

$$= \dfrac{\lim\limits_{x \to 0} \left(\frac{\sin(x)}{x} \right)}{2}$$

Take the numerator. 3

$$\lim_{x \to 0} \left(\frac{sin(x)}{x} \right) = \lim_{x \to 0} \left(\frac{\frac{d}{dx}(sin(x))}{\frac{d}{dx}(x)} \right)$$

$$= \lim_{x \to 0} (cos(x) = 1$$

Plug into the original equation

$$= \frac{1}{2}$$

$$\lim_{x \to 0} \frac{1-cos(x)}{x^2} = \frac{1}{2}$$

b) $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x-4}$

$$= \lim_{x \to 4} \left(\frac{\frac{d}{dx}(\sqrt{x}-2)}{\frac{d}{dx}(x-4)} \right) = \lim_{x \to 4} (\frac{1}{2\sqrt{x}})$$

$$= \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = \frac{1}{4}$$

5. Find the following limits:

a) $\lim\limits_{x \to \pi} \dfrac{sin(x)}{2+cos(x)}$

Substituting the variable with the value:

$$= \frac{sin(\pi)}{2+cos(\pi)}$$

$$= \frac{0}{1}$$

$$\lim_{x \to \pi} \frac{sin(x)}{2+cos(x)} = 0$$

b) $\lim\limits_{x \to 1} \dfrac{\sqrt{2x+1}-\sqrt{3}}{x}$

Substituting x with 1:

$$= \frac{\sqrt{2+1}-\sqrt{3}}{1} = 0$$

$$\lim_{x \to 1} \frac{\sqrt{2x+1} - \sqrt{3}}{x} = 0$$

6. Find the limit:

$$\lim_{x \to \infty} \left(4x - \sqrt{16x^2 - x} \right)$$

$$\lim_{x \to \infty} \left(4x - \sqrt{16x^2 - x} \right) = \lim_{x \to \infty} \frac{x}{4x - \sqrt{16x^2 - x}}$$

$$= \lim_{x \to \infty} \frac{x}{x \cdot \frac{4x - \sqrt{16x^2 - x}}{x}}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{16 - \frac{1}{x}} + 4}$$

Substituting x with ∞:

$$\frac{1}{\infty} = 0$$

Thus $\lim_{x \to \infty} \dfrac{1}{\sqrt{16 - \frac{1}{x}} + 4} = \dfrac{1}{\sqrt{16} + 4} = \dfrac{1}{8}$

$$\lim_{x \to \infty} \left(4x - \sqrt{16x^2 - x} \right) = \frac{1}{8}$$

7. Evaluate the limit: $\lim_{x \to -4} \dfrac{\frac{1}{x} + \frac{1}{4}}{x + 4}$

Applying L'Hopital's rule:

$$\lim_{x \to -4} \frac{\frac{1}{x} + \frac{1}{4}}{x + 4} = \lim_{x \to -4} \frac{\frac{d}{dx}(\frac{1}{x} + \frac{1}{4})}{\frac{d}{dx}(x + 4)}$$

$$= \lim_{x \to -4} \frac{-1}{x^2}$$

Substituting x with -4

$$= \frac{-1}{16}$$

$$\lim_{x \to -4} \frac{\frac{1}{x} + \frac{1}{4}}{x + 4} = \frac{-1}{16}$$

8. Find the limit: $\displaystyle\lim_{x \to 7} \left(\frac{\sqrt{x+2}-3}{x-7} \right)$

$$\lim_{x \to 7} \left(\frac{\sqrt{x+2}-3}{x-7} \right) = \lim_{x \to 7} \left(\frac{\frac{d}{dx}(\sqrt{x+2}-3)}{\frac{d}{dx}(x-7)} \right)$$

Also, $\dfrac{d}{dx}(\sqrt{x+2}-3) = \dfrac{1}{2\sqrt{x+2}}$

Thus $\displaystyle\lim_{x \to 7} \left(\frac{\sqrt{x+2}-3}{x-7} \right) = \lim_{x \to 7} \left(\frac{1}{2\sqrt{x+2}} \right) = \frac{1}{2\sqrt{7+2}} = \frac{1}{6}$

$$\lim_{x \to 7} \left(\frac{\sqrt{x+2}-3}{x-7} \right) = \frac{1}{6}$$

9. Find the limit: $\displaystyle\lim_{x \to 1} \left(\frac{x^3 - 1}{x^2 - 1} \right)$

$$\lim_{x \to 1} \left(\frac{x^3 - 1}{x^2 - 1} \right) = \lim_{x \to 1} \left(\frac{\frac{d}{dx}(x^3 - 1)}{\frac{d}{dx}(x^2 - 1)} \right)$$

$$= \lim_{x \to 1} \left(\frac{3x^2}{2x} \right) = \lim_{x \to 1} \left(\frac{3x}{2} \right)$$

Substituting x with 1:

$$= \frac{3}{2}$$

$$\lim_{x \to 1} \left(\frac{x^3 - 1}{x^2 - 1} \right) = \frac{3}{2}$$

10. Given that f(x) = $\dfrac{1}{x-1}$

Find the instantaneous rate of change.

$$\frac{d}{dx}\left(\frac{1}{x-1} \right) = \frac{-1}{(x-1)^2}$$

The instantaneous rate of change is: f'(x) = $\dfrac{-1}{(x-1)^2}$